# Mean Differences and Inclusion of Uncertainty

**Application Note 6**

**Allan Fraser**

If mean differences are to be assessed then it is necessary to consider the difference in the context of the uncertainties of the values being compared. According to Barwick & Pritchard (2011) and Carr, 2011 differences between means is assessed using [1]:

$$(A - B)^2 < (uA)^2 + (uB)^2 \qquad [1]$$

Where, A and B are two different laboratory means for the same analyte and sample, $uA$ is the standard uncertainty of the mean of laboratory A and $uB$ the mean of laboratory B. The standard uncertainties are calculated from the expanded uncertainties for the determination of the analyte by the respective laboratories. The mean results of Laboratory A and B are *in agreement* if the left side term is < the right hand term.

Moreover, the Eurachem *"Terminology in Analytical Measurement, Introduction to International Vocabulary in Metrology"* (2011), states that the standard measurement

uncertainty of the difference between two measurement results is the root mean squares of the standard uncertainties of the two means:

$$u_d = \sqrt{u_1^2 + u_1^2} \quad [2]$$

If the difference between the two mean values, $d<ku_d$ where $k$ is a coverage factor, then the two means are considered to be *meteorologically compatible* (Barwick & Pritchard, 2011).

Example:
Laboratory A reports a mean of 86.78% and Laboratory B reports a mean of 86.82%.
Through method validation, the laboratories have estimated their expanded uncertainty (U) at 95% as ±0.5% and ±0.7%, respectively.

The *U* needs to be converted to *u*, or standard uncertainty for each laboratory:

Laboratory A: $u = \frac{U}{2} = \frac{0.5}{2} = 0.25$

Laboratory B: $u = \frac{U}{2} = \frac{0.7}{2} = 0.35$

Substituting into [1]:

$$(86.82 - 86.78)^2 \leq (0.25)^2 + (0.35)^2$$
$$(0.04000)^2 \leq (0.0625) + (0.1225)$$
$$(0.04000)^2 \leq 0.185$$
$$0.0016 \leq 0.185$$

Since

$$(A - B)^2 < (uA)^2 + (uB)^2$$

The two means are shown to be in agreement.

**References:**

Barwick V.J., Pritchard E. (Eds). (2011). Eurachem Guide; Terminology in Analytical Measurement – Introduction to VIM 3 (2011). ISBN 978-0-948926-29-7.

Carr R.H. (2011). Estimating errors using graphs and taking good data. California State University Los Angeles. March 2011. 14. http://web.calstatela.edu/faculty/kaniol/Err-Gph-Meas-IBooklet.pdf. Retrieved: 29 August, 2016.